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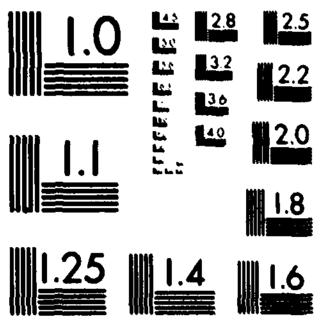

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TIME-FREQUENCY HOP CODING BASED UPON THE THEORY OF LINEAR CONGRUENCES

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Time-frequency hop codes are developed based upon the theory of linear congruences. These codes can be used for multiuser radar and asynchronous spread-spectrum communications systems. A uniform upper bound is placed on the crosscorrelation function between any two elements of the code set. The upper bound is minimized by choice of BT-product and is shown to diminish as 2/N, where N is the number of elements in the code set. The size and position of spurious peaks in the autocorrelation functions are discussed. The results are extended to narrowband ambiguity functions.

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ABSTRACT

Time-frequency hop codes are developed based upon the theory of linear congruences. These codes can be used for multiuser radar and asynchronous spread-spectrum communications systems. A uniform upper bound is placed on the crosscorrelation function between any two elements of the code set. The upper bound is minimized by choice of BT-product and is shown to diminish as 2/N, where N is the number of elements in the code set. The size and position of spurious peaks in the auto-correlation functions are discussed. The results are extended to narrowband ambiguity functions.

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I. INTRODUCTION

In this paper, we investigate a sequence of time-frequency hop codes which can be used in the areas of coherent multiuser radar and asynchronous spread spectrum communications. Unlike synchronous communication systems in which one usually searches for code sets which have mutually orthogonal elements,[1] these applications require that the entire crosscorrelation function between any two elements of the set be small. This is true since the exact time of arrival of the received signal is unknown. Thus, the output of the matched filter for the correct signal must be small when any, or perhaps all, of the other codes are present within the received signal.

In addition to the bound that must be placed on the crosscorrelation functions in these applications, several other constraints may also be placed upon the code set. If time-of-arrival (i.e., range measurement or resolution of closely spaced targets) is of importance, then the autocorrelation functions for the code words must have pulse compression properties.[2] Thus, they must possess a narrow main lobe and small spurious peaks away from the main lobe. If the transmitter, target or receiver is in motion, one must then deal with Doppler effects and, hence, the associated ambiguity functions for the signal set. Thus, one would like the uniform bound placed on the crosscorrelation functions for the code set to extend to the entire crossambiguity function surface. Finally, depending upon whether or not these relative motions are to be measured or ignored determines additional constraints on the autoambiguity function for the code set. Thus, if velocity measurement is to be performed, then the ideal "thumb tack" ambiguity function would be desired.[3] However, should relative velocity be unimportant, then the codes should possess Doppler tolerant[4] properties (i.e., a "razor blade" autoambiguity function with minimum range-Doppler coupling).

The code set presented here, whose structure is based upon the theory of linear congruences, possesses most of the desirable properties described above. Using the number-theoretic properties of linear congruences, we provide an upper bound on the crosscorrelation function for any two elements of the code set. Further, we find an expression relating the time-bandwidth product occupied by the code set to the number of code words within the set which minimizes the crosscorrelation upper bound. Here bandwidth is defined as the range of sinusoidal frequencies spanned by the set, ignoring the spectral skirts. Since all elements of the code set span all available frequencies, each code word possesses maximum pulse compression capabilities. A bound is also placed on the amplitude of the spurious peaks in the autocorrelation function showing the trade-offs involved in the choice of the time-bandwidth product, the code set size, and the amplitude of the spurious peaks. In addition, the positions of the major spurious peaks are also predictable.

Finally, since the code words have virtually identical time delay and frequency shift properties, these results extend directly to the narrow-band crossambiguity functions for the code set.

II. THE CODE WORD SETS

We consider a rectangular pulse of length T seconds, divided into N equal segments of length T/N seconds. For the sequel, N will be restricted to be a prime number. We place in each segment of the pulse a cosine wave whose frequency is one of the N frequencies defined by

$$\omega_k = \omega_o + \frac{kB}{N}, \qquad k = 0, 1, \ldots, N-1 \tag{1}$$

where B is defined as the approximate bandwidth of the signal. Thus, each code word occupies a time-bandwidth product of approximately 2 BT. We shall assume ω_o to be sufficiently high to assure we are dealing with analytic signals. We place one, and only one frequency, in each time slot by using a linear congruential relationship as follows: consider the linear congruence

$$k(m,\ell) = m\ell (\mathrm{Mod}\ N), \quad 0 \leq m,\ \ell < N \tag{2}$$

where k is the frequency, m is the code word index, ℓ the time slot, and N is a prime number.

As an example, consider the code array generated for N = 5. This is shown in Figure 1. Note that except for the left most column, $\ell = 0$, the array is completely occupied. The number inside each block is the code word index. The code word indexed by m = 0 is a CW pulse and will be dropped from the set. We observe that the code word m = 1 is the same pulse Klauder, et al.[2] used to heuristically derive the chirp radar pulse. Thus, this procedure is a generalization of linear frequency modulation.

k, frequency slot

k \ m	0	1	2	3	4 (m, code word index)
4		4	2	3	1
3		3	4	1	2
2		2	1	4	3
1		1	3	2	4
0	34 012	0	0	0	0
ℓ, time slot	0	1	2	3	4

Figure 1. Time-Frequency Hop Codes for N = 5

In the sequel, we shall first review certain properties of linear congruences in the form of Equation (2) and then, using these properties, establish the crosscorrelation upper bounds.

III. PROPERTIES OF LINEAR CONGRUENCES

In this section, we review the properties of linear congruences. For a complete treatment of the theory of linear congruences, see Griffen[5] from which most of the results presented here are taken.

Two integers are congruent (Mod N) if, and only if, their difference is divisable by $N \neq 0$. The totality of integers congruent to a given integer for modulus N constitutes a residue class (Mod N). Any set of $|N|$ integers selected so that no two of them belong to the same residue class (Mod N) forms a complete residue system (Mod N). Finally, we define the complete residue system (Mod N) which contains the elements of each residue class with smallest absolute magnitude as the minimally complete residue system (Mod N). As shorthand notation, we use RC, CRS, and MCRS, respectively.

As an example, for N = 5, we have

a) 1, 6, and -4 are elements of the same RC
b) {5,1,7,-2,4} is a CRS

and

c) {-2,-1,0,1,2} is the MCRS.

In general, for N prime and greater than 2, the MCRS is

$$\left\{ -\frac{N-1}{2}, -\frac{N-3}{2}, \ldots, -1,0,1,\ldots, \frac{N-3}{2}, \frac{N-1}{2} \right\}.$$

In what follows, we associate k with the frequency slot, ℓ with the time slot, and m with code word index. The following properties hold for N prime:

prop. 1) For each code word ($m \neq 0$), each frequency slot and each time slot occurs once and only once. Thus, both form CRS's.

prop. 2) Each unequal pair of code words ($m_1 \neq m_2$) has one, and only one, intersection Mod N.

Property (2) is valid for any horizontal shift of one or both code words. Thus,

$$k_1(m_1,\ell) = m_1(\ell - b_1) \text{ (Mod N)}$$

and

$$k_2(m_2,\ell) = m_2(\ell - b_2) \text{ (Mod N)} \quad m_1 \neq m_2$$

have one, and only one, intersection for any pair (b_1,b_2). This is also true for any vertical shifts since a horizontal shift of b units is equivalent to a vertical shift of mb units.

prop. 3) For each pair of unequal code words and for any horizontal shift, the frequency and time differences each form a CRS.

prop. 4) For two equal code words and fixed shift, the frequency and time difference belong to the same RC. The frequency and time differences for all possible shifts form a CRS. For a shift of b time slots, all frequencies are mb(Mod N) slot apart.

This completes the review necessary for the sequel.

IV. CORRELATION PROPERTIES

We shall first energy normalize each code word. In each segment of length T/N seconds, we have

$$f_k(t) = Ae^{j(\omega_k t + \theta_k)}, \quad 0 \leq t \leq T/N \tag{3}$$

The energy of each segment is

$$E_k = \int_0^{T/N} f_k(t) f_k^*(t)dt = A^2 T/N$$

Thus, since there are N segments, we have

$$E = \sum_{k=1}^{N} E_k = A^2 T$$

but $E = 1$ thus $A = 1/\sqrt{T}$. Thus, the rectangular pulse has amplitude $1/\sqrt{T}$.

Now we calculate the maximum crosscorrelation between two segments

$$C_{mn} = \frac{1}{T} \int_0^{T/N} e^{j(\omega_m - \omega_n)t} e^{j(\theta_m - \theta_n)} dt$$

$$= \frac{1}{T} \frac{e^{j(\omega_m - \omega_n)T/N} - 1}{\omega_m - \omega_n} e^{j(\theta_m - \theta_n)}$$

Taking absolute magnitudes and observing that the numerator must be no larger than 2, we have

$$|Cmn| = \begin{cases} \dfrac{2}{T|\omega_m - \omega_n|} , & m \neq n \\ 1/N , & m = n \end{cases}$$

Recall that

$$\omega_m = \omega_o + \frac{mB}{N} , \quad m = 0,1,\ldots,N - 1$$

Thus

$$\omega_m - \omega_n = \frac{(m - n)B}{N}$$

so that

$$|Cmn| \leq \begin{cases} \dfrac{2N}{T|m - n|B} , & m \neq n \\ 1/N , & m = n \end{cases} \qquad (4)$$

In order to combine the results to find the upper bound for the cross-correlation function, we refer to Figure 2 in which two code words are shown with an arbitrary delay.

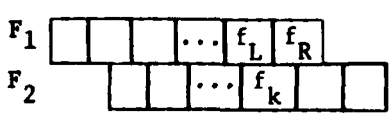

Figure 2. Computation of Cross-Correlation of Two Code Words

Suppose the lower pulse is a reference pulse. Each frequency slot will, in general, multiply two slots in the upper pulse. Thus, there will be contributions from the left portions (f_L's) and right portions (f_R's). We consider these separately. First by property (2), if $f_k = f_L$, we have $\frac{1}{N}$ times the overlapping time, T_o. Obviously since there will also be one other position for which a different slot will match frequencies with right portions, and that contribution is $T/N - T_o$ times $\frac{1}{N}$, the total contribution is $\frac{1}{N}$. For each of the other frequencies, by property (3), the difference will form a CRS. An upper bound is clearly obtained if we choose the MCRS since this is the worst case. Taking into consideration left and right segments, we have that

$$|C_{cc}| \leq \frac{1}{N} + 2 \sum_{\substack{k=-\frac{N-1}{2} \\ k \neq 0}}^{\frac{N-1}{2}} \frac{2N}{T|k|B}$$

or

$$|C_{cc}| \leq \frac{1}{N} + \frac{8N}{BT} \sum_{k=1}^{\frac{N-1}{2}} \frac{1}{k}$$

Thus

$$C_{UB} = \frac{1}{N} + \frac{8N}{BT} \sum_{k=1}^{\frac{N-1}{2}} \frac{1}{k} \tag{5}$$

The sum in Equation (5) is upper bound by

$$\sum_{k=1}^{\frac{N-1}{2}} \frac{1}{k} \leq 1 + \ln\left(\frac{N-1}{2}\right), \quad N \geq 3$$

so that as a further bound we have

$$C(N) = \frac{1}{N} + \frac{8N}{BT}\left[1 + \ell n\left(\frac{N-1}{2}\right)\right] \tag{6}$$

Figure 3 shows a family of curves for C(N) parameterized by BT. Although these curves are shown for all N, they are only valid for N prime. Also, observe that each curve has a value of N for which it is minimum. In order to obtain this we differentiate C(N) with respect to N, set the results equal to zero, and solve for BT yielding

$$(BT)_{opt} = 4N^2\left[\frac{2N-1}{N-1} + \ell n\left(\frac{N-1}{2}\right)\right] \tag{7}$$

which is asymptotically

$$(BT)_{opt} \sim 4N^2\left[2 + \ell n(N/2)\right] \qquad (N \to \infty) \tag{8}$$

Substituting Equation (7) into Equation (6), we obtain

$$C(N)_{min} = \frac{1}{N}\left[1 + \frac{1 + \ell n\left(\frac{N-1}{2}\right)}{\frac{2N-1}{N-1} + \ell n\left(\frac{N-1}{2}\right)}\right] \tag{9}$$

which has an asymptotic expression

$$C(N)_{min} \sim \frac{2}{N}\left[1 - \frac{1/2}{\ell n(N/2)}\right] \leq \frac{2}{N}, \quad (N \to \infty) \tag{10}$$

Thus, we conclude that if BT grows as Equation (8), then the uniform upper bound between all elements of the code set goes to zero; that is, $C(N)_{min} \sim \frac{2}{N}$.

$$A_1 = C(N) = \frac{1}{N} + \frac{8N}{BT}\left[1 + \ln\left(\frac{N-1}{2}\right)\right]$$

$$B = 2\pi F$$

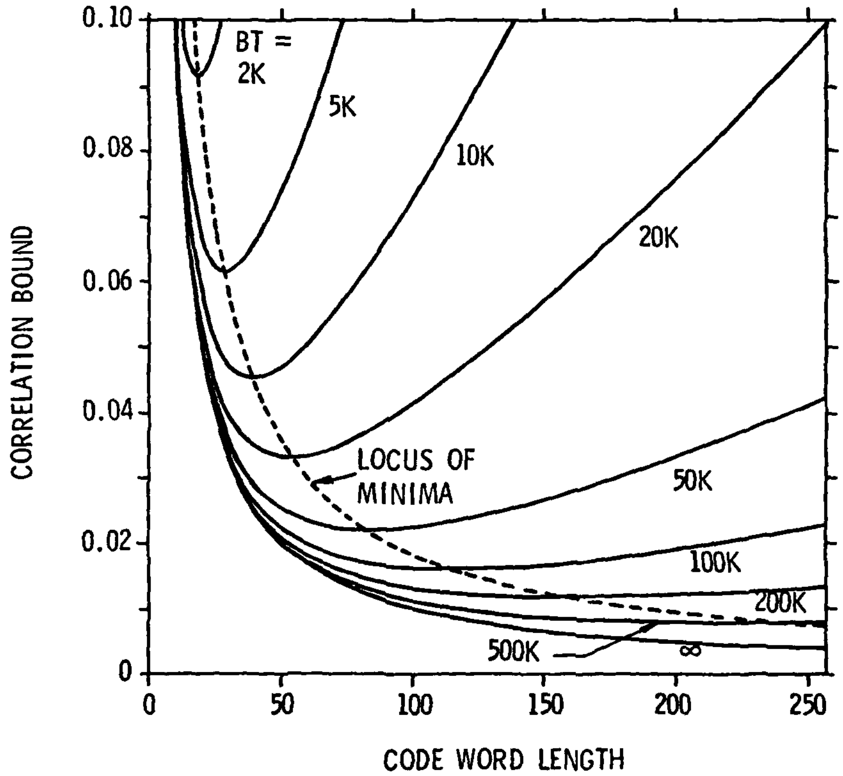

Figure 3. Correlation Bounds of the Code Words

V. AUTOCORRELATION PROPERTIES

Property four indicates that after T/N seconds the frequencies are all 1 slot (Mod N) displaced. Thus, we may bound the autocorrelation by 2N times the correlation for two segments, 1 slot apart. Thus,

$$|A| \leq 2N \left[\frac{2N}{BT}\right] = \frac{4N^2}{TB} \tag{11}$$

and if $(BT)_{opt}$ is chosen from Equation (7) and substituted into Equation (11), we have

$$|A| \leq \frac{4N^2}{4N^2 \left[\frac{2N-1}{N-1} + \ln\left(\frac{N-1}{2}\right)\right]} = \frac{1}{\left[\frac{2N-1}{N-1} + \ln\left(\frac{N-1}{2}\right)\right]} \tag{12}$$

which becomes asymptotically

$$|A| \leq \frac{1}{2 + \ln(N/2)} \tag{13}$$

It is clear that a trade-off is involved. If $(BT)_{opt}$ is chosen, the autocorrelation bound goes to zero very slowly, i.e., as $1/\ln(N)$; whereas, if BT grows faster, say as N^3, then we obtain better sidelobe properties of the autocorrelation but may suffer in the crosscorrelation bound.

VI. CONCLUSIONS AND EXTENSIONS

We have established that, for the class of time frequency hop codes defined here, we can uniformly upper bound the crosscorrelation function between any two elements of the code set. We have shown that the bound can be minimized by appropriate choice of BT product, which insures that the crosscorrelation function bound will diminish as 2/N for N large. It should be noted that the crosscorrelation functions themselves could still remain low since only the bound could be increasing. However, several computer simulations have convinced the author that the bound is generally tight.

In this paper, we have results for finite energy signals only. Yet the inclusion of finite average power signal, defined as periodic extensions of the signals presented, is obvious. With the modifications that

$$R_{fg}(\tau) = \frac{1}{T}\int_{0}^{T} f_1(t)f_2^*(t+\tau)dt$$

and normalization constant in Equation (13) set to 1, the same bounds are valid since the bounding procedure assumed <u>all</u> possible frequency differences were present for <u>all</u> delays.

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